Transportation Theory and Planning of a Traffic Light System

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1 Introduction

The Planning of a Traffic Light System problem is about adjusting the times of green light signals at crossings so that traffic jams are minimized.

The Transportation Theory optimizes the method of transporting goods to minimize the time or the cost of the transport (or, equivalently, to maximize one's profit).

The goal of this work is to present a method for converting the Traffic Light System theory to a Transportation Problem.

2 The conversion

Let:

- D - the set of destination directions (all outgoing),
- S - the set of all source directions,
- N - the number of cars arriving at the crossing in a unit of time.

The resources to be split are the destination directions, and the receivers are the destination directions along with the source directions. Thus, the set of producers is D, and the set of receivers - $S \times D$.

Let:

- the number of producers $m = \overline{\overline{D}}$,
- the number of receivers $n = \overline{\overline{S \times D}}$,

- the quantity of production $A_i = N, \quad i = 1, 2, \ldots, m$ (each destination direction can output any number of cars, e.g. N),
- the quantity of demand $B_j, \quad j = 1, 2, \ldots, n$ is the number of cars arriving a the crossing from the given direction s_k $(1 < k < \overline{\overline{S}})$, wanting to leave in the direction d_l $(1 < l < m)$,

A fake receiver (a "storage"), whose demand shall equal the difference: B_{n+1} between the numbers of arriving and leaving cars, needs to be introduced, allowing us to have a closed Transportation Problem. This models the cars that stayed on the crossing.

The cost function shall be constructed the following way:

$$f(X) = \sum_{i=1}^{m} \sum_{j=1}^{n+1} \Big[\sum_{k=1, k \neq j}^{n+1} \Big(B_k - \sum_{l=1}^{m} x_{lk} \Big) \Big] x_{ij}^2$$

where:

- $X = [x_{ij}]_{i=1,2,\ldots,m; j=1,2,\ldots,n+1}$ - the matrix of cars $d \in D$ assigned to receivers (meaning the amount of transport from the producer to the receiver),

The cost function shall rise the more, the more cars leave the crossing from a given direction and the more cars are left waiting from the other directions (the initial number of provided cars minus the number of those which left already).

Constructing the cost function this way causes results where more cars leave from the direction which provides more, to be preferred, yet limiting situations where cars from only one direction (the one which has most) leave the crossing.

3 Conclusions

The Traffic Light System problem can be converted to a non-linear closed Transportation Problem.